A straight interpretation of the solids flux theory for a three-layer sedimentation model

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Received 10 May 2001; received in revised form 26 September 2001; accepted 23 November 2001

Abstract

One-dimensional settling models based on the conservation law calculate the flux of particles via discrete horizontal layers. Commonly at least 10 of these spatial elements with constant volume are considered in order to achieve a satisfying fit with data. This paper deals with a conceptual approach of a simplified layer model, its theoretical confirmation and its applicability. The model differs three layers with variable volume—clarification-, hindered settling- and compression zone—which are not derived from numerical requirements but from the basic principles of solids flux theory for batch sedimentation. In accordance to the three types of propagation velocities of solids flux theory—hindered settling, signal speed and propagation of concentration discontinuities—three density layers of settling particles of a suspension have been defined. Model development has focused on a reduction of model complexity for on-line control purpose and for coupling with a biokinetic model. Another intention was the model’s universal applicability to both batch and continuous settling in order to compare different types of activated sludge strategies. The model has been evaluated against the analytical solution of the flux function and against data from full-scale SBR experiments and from a secondary clarifier under hydraulic overload. © 2002 Elsevier Science Ltd. All rights reserved.

Keywords: Sedimentation; Solids flux theory; Layer model; Hindered settling; Thickening; Activated sludge modelling

1. Three velocities of solids flux theory

One-dimensional sedimentation models state a constant horizontal concentration distribution of settling particles in a dispersion and hence are addressed as layer models. All commonly applied layer models are based on the sedimentation theory of Kynch [1], the so-called solids flux theory. Besides constant horizontal density layers Kynch assumed another fundamental principle: He declares that the settling velocity of particles at any point in a dispersion depends on the local concentration of particles. The settling process is exclusively defined by the continuity equation regardless of actual forces affecting the particles. Based on this assumption the particle transport (solids flux $f$) due to gravity is expressed by the settling velocity $v_s$ and the particle concentration $X$:

$$v_s = v_s(X),$$
$$f(X) = v_s(X)X.$$  \(1\)

Concentration $X$ terms the mass of particles per unit volume and flux $f$ the mass of particles passing a horizontal section per unit area per unit of time. The continuity equation expresses the relationship between concentration- and flux variation. Considering a column layer with the thickness $dz$ at the distance $z$ from the surface the net flux to this spatial element yields to the alteration of the concentration in this layer:

$$\frac{\partial (X \, dz)}{\partial t} = (f(z) - f(z + dz)) \, dz,$$
$$\frac{\partial X}{\partial t} + \frac{\partial f}{\partial z} = 0.$$  \(2\)
Since the flux depends on $z$ and $t$ the continuity equation (2) can be transformed to a quasi-linear equation:

$$\frac{\partial X}{\partial t} + \frac{\partial f(X)}{\partial X} \frac{\partial X}{\partial z} = 0,$$

$$f = f(X(z, t)).$$

(3)

Eq. (3) describes a straight line in a $z$ against $t$ diagram and is called a characteristic. The physical meaning of the characteristic is the connection of points or levels with a constant particle concentration in the course of the settling process. Consequently, two neighbouring points along a characteristic in the $z-t$ plane have to agree with

$$\frac{\partial X}{\partial z} dz + \frac{\partial X}{\partial t} dt = 0,$$

$$X(z + dz, t + dt) = X(z, t).$$

(4)

Substitution of Eq. (3) into Eq. (4) yields in the slope $dz/dt$ of the characteristic—the signal speed $V(X)$. The signal speed describes the propagation of a region with a plane have to agree with

$$\frac{\partial X}{\partial z} dz + \frac{\partial X}{\partial t} dt = 0,$$

$$X(z + dz, t + dt) = X(z, t).$$

(4)

Beside settling velocity $v_s(X)$ and signal speed $V(X)$ a third velocity $U(X_1, X_2)$ was defined—the propagation velocity of a concentration discontinuity (compare Fig. 3). A concentration step between the particle concentrations $X_1$ and $X_2$ propagates like a wave front during the settling process considering the principle of mass conservation. Applied on a concentration discontinuity rising at the propagation velocity $U$ the flow of settling particles into one side of this level equals the flow out on the other side:

$$U(X_1 - X_2) = f(X_1) - f(X_2),$$

$$U = \frac{f(X_1) - f(X_2)}{X_1 - X_2}.\hspace{1cm}$$

(6)

The relation between the two boundary concentrations $X_1$ and $X_2$ is the so-called jump condition or Rankine–Hugoniot condition. $U(X_1, X_2)$ describes the wave propagation both of discontinuities determined by the initial vertical concentration distribution and discontinuities originating from an immobile interface like the bottom of a settler. Considering a discontinuity as a very small change in particle concentration the difference expression (6) turns to the partial differential equation (5), $U(X)$ converges against $V(X)$, respectively.

The reviewed sedimentation theory addresses universally various fields of application of the conservation law e.g. explains retardation effects associated with traffic jams [2]. Jeppsson [3] with reference to Diehl [4] showed a numerical solution approaching the correct analytical solution of the conservation law as a non-linear differential equation introducing source and sink terms necessary to describe continuous sedimentation. In order to select a unique, physically relevant solution the so-called entropy condition had been imposed as an additional condition.

2. Conceptual three-layer sedimentation model

The most common layer models discretise the settler volume in horizontal layers, each layer assumed to be completely stirred. To each layer a mass balance is applied. The settling mass flow from one layer to the other is limited by the minimum flux of both the considered interface and the interface below. This minimum condition is applied to all layers and allows a simple representation of the effects of boundary conditions like settler bottom, sinks and sources on the pattern of mass flow in the total volume.

Based on this minimum flux condition Vitasovic [5] presented a 10-layer model. This model was adopted by Takács et al. [6] who suggested a double exponential expression representing the settling function—i.e. the relation between settling velocity and particle concentration. This approach considered a reduced settleability of the microfloc fraction of activated sludge and therefore improved the model validity for low solids concentrations. Grijspreert et al. [7] compared and evaluated complexity and data fit of six different layer models and the Takács model achieved the top score in this rating. Ekama et al. [8] gave a comprehensive survey on the development and state of the art of 1-D settler modelling.

Setting the goal of a reduction of model complexity one question remains: How many layers are required for a satisfying model performance. Krebs [9] points out that the numbers of layers should be increased such that the result is not sensitive on this any more. If the result is indeed sensitive on the number of layers, this could serve as a calibration parameter. Other authors [10] suggest 50 layers as an acceptable compromise of model accuracy and computational effort. Obviously, the chosen effort should depend on the case of application. Concerning the numerical approach to the analytical solution of the conservation equation the accuracy of the results is not affected by the number of layers, but only the resolution of the results [3]. The complexity and computational effort of this approach is caused by the applied algorithm.
The concept of the three-layer model [11] which will be discussed in this paper aims at a reduced complexity in order to couple the sedimentation model with a biokinetic model. Moreover, a simple layer structure appears to be more suitable for further modifications necessary for the modelling of cyclic activated sludge processes [12,13]. As mentioned above the steps of spatial discretisation cannot be arbitrarily reduced without loss of model accuracy. In order to minimise the number of layers those are not considered as constant numerical elements but as physically relevant part-volumes. Defined interfaces between the chosen layers should correspond with the observed discontinuities in particle concentration. Measured concentration profiles during batch-sedimentation show three characteristic layers with variable volumes. Variable layer volumes mean that the levels of the interfaces of the layers are calculated explicitly in a mechanistic or conceptual way and not approximated within the structure of a fixed spatial discretisation.

The top layer between the water level and sludge blanket is called the supernatant water layer. In the settling zone below the sludge blanket hindered settling takes place. During a batch-sedimentation process a zone of increased concentration rises from the bottom of the tank where compression dominates settling. Batch-sedimentation ends with a stable equilibrium when the settling zone has sunk into the compression zone.

The following conceptual description of the settling process has been derived from the observed development of layers during batch-sedimentation (Fig. 1): Starting from a constant vertical concentration profile the settling sludge body descends continuously with constant concentration if no influent flow as a source term is considered. Near the bottom the settling velocity approaches zero. Hence, the settling zone sinks continuously into the compression layer. This compression layer shows a simultaneous increase of volume and concentration and both growth effects are fed by the flux entering from the settling layer. A certain fraction of the entering flux compensates the concentration difference between the settling layer and the compression layer in the interface zone. The other fraction of the flux contributes to the increase of the concentration in the compression layer. The fractionation of the mass flow from the settling layer to the compression layer for layer growth and concentration increase is determined by a calibrated flux distribution coefficient $f_c$ which depends on the sludge volume index and the sludge concentration in the settling layer (see the appendix, [11]).

Each time step comprises two calculation steps: The first step calculates the new layer heights $H_1$, $H_2$ and $H_3$ out of the settling velocities, flux and concentration differences ($T = 1a$ in Fig. 1). The second step calculates the new concentrations $X_1$, $X_2$ and $X_3$ out of the new layer heights, respectively, the volumes and the flux ($T = 1b$ in Fig. 1).

The particle concentration in the supernatant water layer which equals the effluent quality can be determined by an empirical approach e.g. from Busby and Andrews [14]. These approaches usually consist of one term considering a minimum fraction of unsettleable particles and one term for the influence of the surface load. Generally, a layer model is not the appropriate tool to calculate solids effluent concentrations [15]. Only two-phase 3-D models are able to simulate various hydraulic effects in a settler [16], which influence the particle concentration in the supernatant water. If the model application aims primarily at biological processes or sludge storage mechanisms the particle concentration in the supernatant water layer can be neglected and consequently the biokinetic processes are restricted to two reactor volumes. At the beginning of a batch-sedimentation process the concentration discontinuity between settling layer and compression layer is initialised and the compression layer starts from a zero volume. The equations for the calculation of the layer heights and concentrations are strictly based on this presented concept, on the principles of solids flux theory and therefore on mass conservation. Since each layer is
assumed as a completely stirred zone any external influent or effluent flow is balanced as a source or sink within the corresponding layer.

3. Theoretical confirmation of the conceptual model approach

In this chapter the three velocities defined by the solids flux theory, i.e. settling velocity, signal speed and propagation velocity of concentration discontinuities, are applied in order to investigate the conceptual approaches for the most simple settling process—batch-sedimentation.

The settling velocity \(v_s\) depending on the particle concentration usually is described by an exponential expression including two fit parameters \(v_{s0}\) and \(n\) in Eq. (7) [17]. In order to reduce the calibration effort the fit parameters can be related to the key parameter of sludge settling—the sludge volume index. In literature such expressions have been parameterised [18,19] (see the appendix).

\[
v_s = v_{s0} e^{-nX}.
\]

Since the settling velocity \(v_s\) of the sludge blanket is the same as the velocity of the particles directly below, this velocity is determined by Eq. (7). Starting from an initial particle concentration \(X_0\) the settling layer including the sludge blanket descends with the velocity \(v_s(X_0)\). This calculation can be confirmed by a different consideration: The sludge blanket represents a concentration discontinuity. The suspended solids concentration above the sludge blanket is zero or close to zero and below the sludge blanket \(X_0\). Hence the propagation velocity \(U(X_1, X_2)\) of the sludge blanket satisfies Eq. (6)

\[
U = \frac{f(X) - f(0)}{X_0 - 0} = \frac{v_s(X_0)X}{X_0} = v_s(X_0).
\]

To calculate the propagation of the discontinuity rising from the bottom the concentration directly below the interface needs to be investigated. Two conditions can be applied for this purpose: On the one hand, the suspended solids against time profile below the density step satisfies Eq. (5) describing the signal speed \(V\). On the other hand, the third velocity \(U\) of Eq. (6) expresses the propagation of the discontinuity. Combining the two equations leads to a numerical calculation of the particle concentration at the layer interface. The graphical representation [3] of the combination of the velocities \(U\) and \(V\) shows that the straight line connecting the two boundary sludge concentrations \(X_0\) and \(X_{\text{tan}}\) at the settling flux curve is equal to the gradient of the tangent to the curve at the concentration \(X_{\text{tan}}\) (Fig. 2).

\[
U = \frac{f(X_{\text{tan}}) - f(X_0)}{X_{\text{tan}} - X_0} = \frac{\partial f(X_{\text{tan}})}{\partial X} = V(X_{\text{tan}}).
\]

Both the propagation of the sludge blanket and of the discontinuity near the bottom show a linear profile in the \(z-t\) plane. Consequently, the sedimentation process is defined until the two shock fronts clash. When the rising discontinuity reaches the settling sludge blanket the concentration directly below the sludge blanket increases. Hence, the settling velocity of the sludge blanket decreases and the profile in the \(z-t\) plane gets bent after a distinct kink of the curve. The exact profile in this range of the \(z-t\) plane is determined by a consideration of Kynch [1]:

A characteristic represents a level with a constant particle concentration and this level rises (or descends) with the velocity \(V(X)\). Consequently, the settling velocity \(v_s(X)\) of the particles crossing the considered level stays constant during the settling period. A characteristic reaches the surface of a settling sludge body (i.e. the sludge blanket) when all particles initially above the characteristic have passed. This condition can be expressed by the following equation:

\[
X(v_s(X) - V(X))t = \int_0^{z_0} X(z, t = 0)\, dz.
\]

Eq. (10) determines the time period \(t\) from the beginning of the sedimentation process until the characteristic of a certain density reaches the sludge blanket. In other words the position \(z_0\) of the sludge blanket at certain sludge concentrations can be calculated. Since the characteristics within the compression zone start from the bottom, \(z_0\) of these characteristics comprises the total depth of the settler and the integral

![Graphical determination of the concentration continuous curve](image)
on the right-hand side of Eq. (10) comprises the total sludge mass \( z_0 X_0 \) in the settler:

\[
t = \frac{z_0 X_0}{X_{\text{tan}} (v (X_{\text{tan}}) - V (X_{\text{tan}}) t)}.
\]

\[
z = z_0 + \frac{V (X_{\text{tan}}) X_0}{X_{\text{tan}} (v (X_{\text{tan}}) - V (X_{\text{tan}}))}.
\]

Fig. 3 shows the time-dependent sedimentation of a dispersion which is characterised by the flux curve in Fig. 2. Considering the initial sludge concentration \( X_0 = 2.5 \, \text{g l}^{-1} \) Eq. (9) results in a corresponding boundary concentration of \( X_{\text{tan}} = 13.5 \, \text{g l}^{-1} \). The level where this density jump occurs rises towards the sludge blanket. After the two density fronts have met, the sludge concentration directly below the sludge blanket increases continuously. Each point of intersection of the sludge blanket with one of the characteristics in Fig. 3 represents an increase of \( 1 \, \text{g l}^{-1} \). The centric formation of the characteristics points at a theoretically unlimited growth of density \( X \) near the bottom.

Fig. 3 illustrates as to where the three-layer approach deviates from the analytical solution: The clarification and the settling layer exhibit homogenous particle densities during batch-settling but the compression layer shows a gradual concentration increase towards the bottom. In order to avoid an unlimited number of compression layers an average solids concentration \( X_3 \) for one homogenous compression layer needs to be calculated. Of course the average concentration \( X_3 \) is not equal to \( X_{\text{tan}} \) and the flux \( f (X_3) \) becomes zero because of the fixed bottom boundary. Inserting both variables into Eq. (6) to calculate the propagation velocity \( U (X_2, X_3) \) of the layer interface requires a correction factor to adapt the approach either to measurement or to the analytical solution. Building the bridge between this simplified approach and the conceptual model the applied correction factor corresponds to the flux distribution coefficient \( f_c \) (appendix):

\[
U (X_2, X_3) = f_c f (X_2) - f (X_3) = f_c f (X_2) = \frac{dH_3}{dt}.
\]

The conceptual model calculates density and volume increase of the compression layer from a constant distribution of the flux from the settling layer to the compression zone. Therefore, the model simulation ends when the settling layer has disappeared. Figs. 4 and 5 compare the calculated results with measurements and the analytical solution. The batch-settimentation tests have been conducted in a 4 m high Plexi-glass pipe with a diameter of 15 cm and four valves at different levels [20].

As another example the sedimentation of a different sludge is described which shows a completely different settling behaviour. Both suspended solids concentration \( X_0 \) and sludge volume index SVI are more than twice as high as in the example before. In this case Eq. (9) offers no solution. The graphical representation of this problem reveals that there is no sludge concentration \( X_{\text{tan}} \) with a tangent to the flux curve crossing the flux curve at the point \( X_0 \) (Fig. 6). The condition that \( U = V \) in Eq. (9) is only satisfied in case that the concentration \( X_{\text{tan}} \) is equal to the initial concentration \( X_0 \). Consequently the characteristic of the density \( X_0 \) rising from the bottom forms the boundary of the compression zone.

As expected the analytical solution presented in Figs. 7 and 8 shows a steady profile of the descending sludge blanket. Because of the smooth concentration
increase at the rising front without density jump the crossing with the descending front affects no kink in the settling profile of the sludge blanket. The slower decrease of the sludge blanket indicates a distinct compression effect which is dampened by the continuously growing density of the compression layer. In this case the model behaviour shows the most obvious deviation from the analytical solution because an average concentration within the compression layer is calculated and the simulated settling process stagnates as soon as no further flux occurs.

4. Model application

The presented three-layer settling model has been developed from batch-sedimentation tests and, therefore, its application for modelling of cyclic activated sludge systems (e.g. SBR procedure) is at primary value. The sedimentation process starts from a completely stirred state and its transient settling process usually
ends before reaching a stable state. The next aeration or stirring phase of the operation cycle causes a reset of the state after sedimentation to the initial completely stirred state. Hence the total sludge mass in a “cyclic settler” is much higher than in a continuous settler. That is the reason why biological processes should not be neglected. Physical transport needs to be coupled with biokinetic reaction.

The transport module addresses internal and external flows. Since each sedimentation layer is considered as a completely stirred reactor the flow from one layer to the other requires formally the same mathematical abstractions as external flows. The transport of particulate and soluble compounds must be considered differently: Sedimentation is a gravity-driven transport of particles relative to the liquid. Additionally, particles are transported by advective flows. Soluble compounds are transported exclusively by advective flows in or out of each reactor with variable volume.

The flow pattern resulting from the discharge of supernatant water are obvious: Discharge can only happen in case the sludge blanket is at least 25 cm below the discharge facilities. The model calculates a reduction of the volume of the supernatant water layer. Settling and compression layer are affected by the discharge only in case of a simultaneous influent flow (see Fig. 9). Such an operation scheme means a displacement of supernatant water by the influent flow at a constant water level in the reactor during the whole operation cycle. The question to which model layer the influent flow should be fed does not only depend on the actual situation of the inlet structure. If the influent flow is fed...
to the reactor near the bottom the added mixed liquid tends to rise because of the density difference between unsettled and settled sludge. A balanced distribution of the influent flow to the settling and the compression layer obtained the best fit to data of the following simulation example.

The model is based on mass conservation and mass balances of all relevant compounds. Besides transport processes the mass balance is affected by transformation processes. Biological reactions are attributed to zones with relevant sludge concentrations, i.e. the two sedimentation layers below the sludge blanket. Biological degradation processes have been calculated by the reaction module according to the Activated Sludge Model No. 1 [21].

Fig. 9 presents simulation results of single tank experiments [22] at the WWTP Strass. The measurement values have been collected during a period when the pilot plant was operated at a constant water level (29.3.1995). The reactor with a volume of 540m³ was 4m deep. The operation cycle of 3h was composed of a mixing phase of 30min, an aeration phase of 60min, a settling phase of 30min and a discharge phase of 60min. During the discharge phase the influent flow was pumped to the reactor at a constant rate of 23l s⁻¹ and supernatant water was displaced.

The calibration example in Fig. 9 presents results from a period of 1 day divided into eight cycles with three cycles of dense sampling. Ammonia influent concentrations have been measured hourly in composite samples and the other data were based on grab samples. The simulated sludge blanket profile indicates the beginning of the settling phase in every cycle when it deviates from the constant water level. At the beginning of the discharge phase the sludge blanket profile shows a kink. The hydraulic load during the discharge phase causes initially a reduction of the settling velocity and then a steady increase due to the dilution of the sludge concentration in the settling layer. At the beginning of the mixing phase the sludge blanket rises immediately back to the surface.

The mixing of the total reactor volume levels out the nitrogen concentrations of each separate layer. This effect creates step changes of the ammonia and nitrate profiles as illustrated for the supernatant water layer in Fig. 9: The accumulated ammonia from the influent flow causes an ammonia increase while the nitrate drops when nitrate-free sludge zones are mixed with the supernatant water layer. During the aeration period ammonia is oxidised to nitrate and consequently both graphs show opposite trends. During the anoxic settling and discharge phases the nitrate concentration in the supernatant water layer decreases although almost no active biomass is available in this zone. The decrease of nitrate in the clarification layer is associated with the transport of soluble compounds from the biologically active settling and compression layers as described above. In this way the nitrogen concentration profiles characterise not only biological nitrogen removal processes but also the shifting of the sludge layers.

In order to explore the limits of the applicability of the model it was employed for the simulation of secondary clarification. The batch-sedimentation model was adapted to a continuous clarifier model at the same stage of simplicity: Entropy conditions and the
calculation of effluent solids concentrations are still neglected. The influent flow is fed to the model's sludge layers as source terms \( Q_1, Q_2, Q_3 \) and therefore affects their volumes and sludge concentrations. Return flow \( Q_r \) and effluent flow \( Q_{ef} \) are defined as sinks. The external net flows \((Q_1 - Q_{ef}), (Q_2 - Q_1)\) contribute as advective components to the total settling process (compare the calculation of the layer heights in the appendix).

The presented model approaches have been applied to simulate the circular clarifier of the WWTP Grafing [23]. Simulation results of the sludge blanket height could sufficiently match detailed monitoring data of the clarifier performance under hydraulic overload. Still the models intention never was the representation of complex hydraulic effects. The simple model structure is most suitable for a clear subdivision of a settling tank into different zones of sludge concentration or reaction volumes, respectively.

5. Conclusions

Fifty years ago Kynch [1] presented his “Theory of Sedimentation”. Nowadays, in the era of three-dimensional numerical modelling of flow and transport of dispersions, the solids flux theory is still an often cited simple approach towards a complex issue. The theory’s three velocities—hindered settling velocity, signal speed and propagation velocity of concentration discontinuities—provide the mechanistic background of the presented three-layer batch-sedimentation concept. A straight interpretation allows a simple model structure for specific model applications:

- The compression layer rising from the bottom boundary with unlimited density is represented by a CSTR with the average solids concentration of the propagating density front.
- The empirical coefficient \( f_c \) describes the distribution of the settling flux which enters the compression zone to feed the volume and the concentration increase, respectively.
- Especially for cyclic settling processes the solids concentration in the supernatant water layer can be neglected.
- The model gives a satisfactory description of sludge blanket variations and sludge retention—two major design and operation criteria of cyclic and of continuous activated sludge systems.
- Coupling of each sludge layer with a biokinetic model enables the calculation of denitrification during settling, which is a relevant issue especially for cyclic activated sludge systems.

### Appendix

Calculation of layer heights \( H \) and sludge concentrations \( X \) [11]:

\[
\frac{dH_1}{dt} = \frac{Q_m - Q_{ef} - Q_r}{a}, \quad H_1 = H_1' + dH_1 - dH_2,
\]

\[
\frac{dH_2}{dt} = \frac{Q_r}{a} \left( 1 - \frac{X_1}{X_2} \right) + \frac{Q_1 - Q_r}{a} - v_s, \quad H_2 = H_2' + dH_2 - dH_3,
\]

\[
\frac{dH_3}{dt} = f_c v_s X_2 \frac{Q_1 - Q_r}{X_3 - X_2} + \frac{Q_3 - Q_r}{a}, \quad H_3 = H_3' + dH_3.
\]

\[
X_1 = 0,
\]

\[
X_2 = \left( (Q_1 + Q_2)X_m - \left( av_s + Q_r + \frac{a dH_3}{dt} \right) X_2' \right) \frac{dt}{aH_2 + X_2'H_2'},
\]

\[
X_3 = \frac{(X_m Q_m - X_1 Q_{ef} - X_3 Q_r) dt}{a}
\]

\[
+ \frac{X_1'H_1' + X_2'H_2' + X_3'H_3' - X_1 H_1 - X_2 H_2}{H_3}.
\]

\[
v_s = 187 e^{(-0.148 - 0.0021 SVI) X_2}, \quad \text{...gravitational settling velocity [19]}\]

\[
\frac{f_c}{20250 - (Q_r/a)^{1/3}} 3425, \quad \text{...flux distribution coefficient.}
\]

\[
dt = 5 \text{ min}, \quad \text{...calculation time step.}
\]

\[
Q_m = Q_1 + Q_2 + Q_3, \quad \text{...influent flow rate.}
\]

\[
Q_{ef}, \quad \text{...effluent flow rate.}
\]

\[
Q_r, \quad \text{...return sludge flow rate.}
\]

\[a, \quad \text{...settler surface area.}\]
References